Bernoulli Distribution: Bern(p)

Motivation: The Bernoulli distribution describes the outcomes of a single trial with two outcomes, success or failure, where p = probability of success.

Definition: If 1 = success and 0 = failure, then X ~ Bern(p) when

$$Rng(X) = \{0, 1\}$$
$$f(X) = \{1 - p, p\}$$

$$E(X) = p$$
$$Var(X) = (1 - p)p$$

Binomial Distribution: B(N,p)

Motivation: The binomial distribution describes the number of successes which occur among N independent Bernoulli trials.

Definition: $X \sim B(N, p)$ when

$$Rng(X) = \{0, \ldots, N\}$$
$$f(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$E(X) = Np$$
$$Var(X) = N(1 - p)p$$

Geometrical Distribution: G(p)

Motivation: This counts the number of Bernoulli trials until the first success occurs.

Definition and Example: X ~ G(p) if

$$Rng(X) = \{1, 2, \ldots\}$$
$$f(k) = (1 - p)^{k-1} p$$

$$E(X) = \frac{1}{p}$$
$$Var(X) = \frac{1 - p}{p^2}$$

$$P(X > k) = (1 - p)^k$$
$$P(X \le k) = 1.0 - (1 - p)^k$$

Negative Binomial Distribution: NB(r, p)

Motivation: This is a generalization of the Geometric: it counts the number of Bernoulli trials until the r^{th} success occurs.

Definition and Example: $X \sim NB(r, p)$ if

$$Rng(X) = \{r, r + 1, \ldots\}$$
$$f(k) = \binom{k - 1}{r - 1} (1 - p)^{k-r} p^r$$
$$E(X) = \frac{r}{p}$$
$$Var(X) = \frac{r(1 - p)}{p^2}$$

Poisson Distribution: Poi(λ)

Motivation: If we have a process in which events arrive (hence, called arrivals) independently through time, with some mean rate λ = # arrivals/unit time, then the Poisson characterizes how many arrivals will occur in a randomly chosen time unit.

Definition: X ~ Poi(λ) if

$$Rng(X) = \{0, 1, 2, \ldots\}$$
$$f(k) = \frac{e^{-\lambda} \cdot \lambda^k}{k\,!}$$

$$E(X) = \lambda$$
$$Var(X) = \lambda$$

where $e = 2.71828183\ldots$ (Euler's constant).

Exponential Distribution: Exp(λ)

Motivation: If we have a process in which events arrive (hence, called *arrivals*) independently through time, wit the Exponential characterizes the inter-arrival time, e.g., "how long until the next arrival"?

Definition: X ~ Exp(λ) if

$$Rng(X) = [0, \infty)$$
$$f(t) = \lambda\, e^{-\lambda t}$$
$$F(t) = 1.0 - e^{-\lambda t}$$

$$E(X) = \frac{1}{\lambda}$$
$$Var(X) = \frac{1}{\lambda^2}$$

$$P(X > t) = e^{-\lambda t}$$
$$P(X \le t) = 1.0 - e^{-\lambda t}$$

where $e = 2.71828183\ldots$ (Euler's constant).

Normal Distribution: N(μ, σ^2)

Motivation: The normal is the limiting case of the binomial $B(N, 1/2)$ as $N \to \infty$, parameterized with respect to its mean and variance. It describes the distribution of data expressed as real numbers which exhibits the "bell-shaped curve"; this describes a wide variety of phenomena such as in biostatistics (height, weight, dimension of various body parts, intelligence, etc.), errors in measurement, etc.

Definition: $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ the standard deviation, if

$$Rng(X) = (-\infty, \infty)$$
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$
$$F(x) = \frac{1}{2}\left[1 + erf\left(\frac{x - \mu}{\sigma\sqrt{2}}\right)\right]$$

$$E(X) = \mu$$
$$Var(X) = \sigma^2$$

Example set of 52 poker playing cards